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# Should the carbon price be the same in all countries?

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## Abstract

International differences in fuel taxation are huge, and may be justified by different local negative externalities that taxes must correct, as well as by different preferences for public spending. In this context, should a worldwide unique carbon tax be added to these local taxes to correct the global warming externality? We address this question in a second best framework à la Ramsey, where public goods have to be financed through distortionary taxation and the cost of public funds has to be weighted against the utility of public goods. We show that when lump-sum transfers between countries are allowed for, the second best tax on the polluting good may be decomposed into three parts: one, country specific, dealing with the local negative externality, a second one, country specific, dealing with the cost of public funds, and a third one, global, dealing with the global externality and which can be interpreted as the carbon price. Our main contribution is to show that the uniqueness of the carbon price should still hold in this second best framework. Nevertheless, if lump-sum transfers between governments are impossible to implement, international differentiation of the carbon price is the only way to take care of equity concerns.

# 1 Introduction<sup>1</sup>

The virtues of a unique carbon price, taking the form of a world carbon tax or a world emission permits market, are well recognized. A unique carbon price, reflecting the true social cost of emissions, is the best incentive to curb all the negative externalities associated with fossil fuel consumption and global warming. Uniqueness of the price implies the equalization of marginal abatement costs and therefore minimizes the worldwide cost of abatement of emissions. The redistribution of tax receipts or the initial allocation of permits offers then the possibility to accompany carbon taxation with an international redistribution scheme and to share equitably the burden of taxation between countries.

This optimistic picture is often questioned in the name of realism. Existing differences in national energy taxation, especially fuel taxation, are important (see Table 1). The question that arises is whether we should consider them as implicit carbon taxes and therefore abstain from super-imposing a specific carbon tax.

Table 1: Excise on unleaded gasoline in some OECD countries, 2008 (% of the consumer price)

|         |         |               |         |        |       |       |                |        |
|---------|---------|---------------|---------|--------|-------|-------|----------------|--------|
| Germany | Denmark | United States | Finland | France | Italy | Japan | United Kingdom | Sweden |
| 46.7    | 38.7    | 14.5          | 42.9    | 43.6   | 40.9  | 34.2  | 44.5           | 40.6   |

Source: IEA Statistics, Energy prices and taxes, 2009

Moreover, is it really possible to use the allocation of tax receipts or emission quotas to alter substantially the world distribution of income? First of all, using a quota allocation mechanism clearly implies that the implied lump-sum transfers to local governments are restricted to be positive. Second, will governments even accept to depart from the simple rule that each country should be paid back exactly the amount of taxes it pays or the value of the permits it had to buy? If international transfers are so restricted, isn't it preferable to allow poor countries to face a lower carbon price? Chichilnisky and Heal [1994] put forward such an argument against the international

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equalization of abatement costs and suggested that a lower effort should be requested from poor countries. Shiell [2003] follows on this idea and in particular characterizes the set of second best optimal allocations when intercountry transfers are restricted to be positive or to exactly equal the sums paid to the world regulator.

A proper analysis of these issues requires a clarification of the purpose of existing fuel taxes in the first place. If no clear economic reason may be invoked for their existence, adding to them a unique carbon price has no chance to appear optimal. To the contrary, if existing taxes are, in some sense, already fixed at an optimum level, adding a common tax to curb global warming may make sense. Existing fuel taxation has two prime objectives. The first one is to counter local externalities independently of global warming. For example, burning of fossil fuel contributes to global warming through  $\text{CO}_2$ , but also leads to  $\text{SO}_2$  emissions which have a more local effect. The second objective of fuel taxation is that it offers governments an easy way to levy funds and finance the provision of public goods. The French gasoline tax, the so-called TIPP (Taxe Intérieure sur les Produits Pétroliers), an excise tax currently fixed at 0.69 euros per liter, has indeed been created explicitly with the two objectives of reducing negative externalities associated with fuel consumption by cars and of providing means to finance highway construction.

International differences in fuel taxation may a priori be justified by different local negative externalities that taxes must correct, as well as by different preferences for public spending in the different countries. In this context, should a worldwide unique carbon tax, or emission permit price, be added to these optimal local taxes? This is the question we consider.

Addressing seriously the financing of public goods provision imposes a second best approach. Public goods have to be financed through distortionary taxation and the cost of these distortions, i.e., the cost of public funds, has to be weighted against the utility of public goods. On the other hand, Pigovian taxes, aiming at reducing negative externalities, and (distortionary) taxes, aiming at financing the provision of public goods, are to some extent substitutes. A Pigovian tax required to decrease emissions is also a means to finance public spending. Inversely, a negative externality associated with the production of a commodity decreases the cost of public funds associated with the use of this good as a tax-base. The negative impact of the distortions associated with taxation are mitigated by the reduction of the negative externality. Sandmo [1975] initiated this kind of analysis in a one country case.

In this paper, we extend the Chichilnisky and Heal [1994] model by introducing local externalities and public goods. We consider a clean consumption good, a dirty consumption good, source of both local externalities and global warming, and a public good. The three goods may be produced in each country from a given endowment of a generic good. There is no international trade and countries interact only through global emissions.

We first examine first best optimal allocations and show that they can be implemented by a world government through country-specific emission taxes and lump-sum transfers to consumers, possibly negative. The pollution tax consists of two parts, one dealing with the local externality and another one dealing with the global externality. The latter part is uniform across countries. This suggests a more decentralized implementation. Local governments then levy a tax correcting the local externality, are in charge of the provision of the local public good, and tax or subsidize their consumers in a lump-sum way. The federal government collects the uniform global pollution tax and makes lump-sum (positive or negative) transfers to individual states, thus taking in charge international equity concerns. This implementation assumes passive local governments, who do not take advantage of the influence they might have on the world carbon price and the levels of international transfers. Basically, this preliminary analysis confirms the optimistic view which we described above. In a first best setting, the uniqueness of the carbon tax should be the rule, provided local governmental decisions are taken in an optimal way.

We then consider a more realistic second best framework. We follow the standard Ramsey approach and assume that governments cannot tax consumers in a lump-sum way. They have to rely if necessary on distortionary taxation to finance public good provision. We show that the second best taxes on the polluting good may be decomposed into three parts: one, country specific, dealing with the local pollution externality, a second one, country specific, dealing with the cost of public funds, and a third one, global, dealing with the global externality. Our main contribution is to show that uniqueness of the carbon price should still hold in this second best framework. This is a striking result as we know since Lipsey and Lancaster [1956–7] that rules which are optimal in a first best world fail to be so in a second best world.

To get a better understanding of this result we show that a true decentralization of the second best optimum may be obtained. The world regulator fixes a common carbon tax as well as lump-sum transfers to local governments. Each of these is then free to deal as it wishes with local externalities

and distortionary taxation needed to finance the provision of local public goods. This result supports, in a somewhat general setting, Tirole [2009]'s proposition to base climate policy upon a world permits market, leaving each country free to choose whatever instrument they want to meet their emission target.

This result relies on the feasibility of arbitrary lump-sum transfers between countries. This takes us back to the Chichilnisky and Heal [1994] analysis<sup>2</sup>. Let us first point out that the issue of the non-negativity of transfers differs when applied to consumers or to local governments.

The impossibility to levy lump-sum taxes on consumers is motivated by information problems. Personalized lump-sum transfers are impossible because governments lack the necessary information about individual characteristics. The Ramsey approach then simply assumes away the possibility to levy a lump-sum tax on consumers. The use of this approach in the case of a representative consumer may be, and has often been, criticized on the grounds that if all agents, in a given country, are identical, nothing prevents the government from levying the same lump-sum tax on all of them. The Ramsey approach may nevertheless be considered as a useful short-cut to analyze a situation where distortionary taxation is required. Sandmo [2000] p. 97, among others, provides such a defense.

The impossibility to implement arbitrary lump-sum transfers between governments cannot be motivated by information problems. Political economy reasons are more convincing. Governments know that their citizens would probably object to sizable transfers to other governments. The assumption of no inter-governmental transfers may then appear realistic. The receipts of the world carbon tax, or the product of the sale of the permits, have to be redistributed however. In our setting, the no inter-governmental transfers assumption has to be rephrased as the assumption that each country should receive a positive lump-sum transfer precisely equal to the amount of carbon taxation supported by its citizens. We examine in a last section the second best Pareto optimum under this assumption. The non-negativity constraint on transfers to consumers still holds. We thus revert to the Chichilnisky and Heal [1994] result. International differentiation of the carbon price is the only way to take care of equity concerns. Poorer countries should proba-

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<sup>2</sup>Sheeran [2006] reexamined their argument in a more general setting. Also, Sandmo [2006] shows in a model without public spending and local externalities that correcting a global externality by an internationally uniform Pigovian tax is only optimal when international lump-sum transfers are possible.

bly pay a lower carbon tax. This result may be put in perspective however. First, the redistributive gains of such a differentiation of carbon taxes may be small. A substantial amount of redistribution would then be at the cost of an important loss of efficiency. Second, accepting to implement direct inter-governmental transfers would enlarge the domain of feasible allocations. A step in this direction has been made at the Cancun *United Nations Framework Convention on Climate Change* conference in December 2010. A Green Climate Fund has been established to assist developing countries in the financing of their climate policies. This new institution should receive by 2020 up to 100 billion dollars per year from developed countries.

We restrict our analysis to the case of local governments which are passive in the sense that they take as given the world carbon price and the transfers they receive from the world regulator. Verbon and Withagen [2010] consider on the contrary the case of active local governments who act strategically as they understand that their behavior affects the world carbon price. In a rather general framework, they show that a proper initial allocation of permits may counter strategic behavior and thus lead to a Pareto efficient allocation. Ogawa and Wildasin [2009] follow a different route and show that local government decisions may, in a very special case, produce a Pareto-efficient outcome.

The result that the world carbon tax is uniform brings to mind an important contribution by Gauthier and Laroque [2009], who show that, in certain circumstances, first best rules prevail in a second best world. Our framework however is quite different and their result does not apply in our case. We have altogether different utility functions, whereas their main result, following Atkinson and Stiglitz [1976], assumes identical consumption tastes. Secondly, and more importantly, we follow the Ramsey approach rather than the Mirrlees one. Lastly, we have an international dimension and, in particular, distinguish between transfers to consumers and to countries. Our argument is therefore of a very different kind.

## 2 The model

We consider  $n$  countries ( $n \geq 2$ ), indexed by  $i = 1, 2, \dots, n$ . There is a representative consumer in each. Each country has an endowment  $Y_i$  of a generic good. This good may be used to produce, one for one, a private clean good, a local non-polluting public good and a private polluting



good. Pollution is both local and global and encompasses country specific pollutions and greenhouse emissions contributing to global warming. We denote by  $C_i$ ,  $G_i$  and  $X_i$  consumptions of the three goods by the representative consumer in each country. Pollution is denoted by  $Z$ . The polluting good is scaled in such a way that  $Z_i = X_i$ . World pollution is denoted by  $Z_w = Z_1 + Z_2 + \dots + Z_n = X_1 + X_2 + \dots + X_n = X_w$ . By  $X$  we denote the vector  $(X_1, X_2, \dots, X_n)$ . The vectors  $C$ ,  $Y$ ,  $G$  and  $Z$  are defined in a similar way.

There is no international trade. Each country consumes its production of the three goods and only interacts with others through its contribution to global warming. International transfers of the non-polluting consumption good will be possible, however. As we saw, the production side of the model is highly simplified, all marginal costs of production being basically equal to one, in terms of the generic good. These simplifications do not affect our main results.

The utility function of the representative consumer in jurisdiction  $i$  is

$$U^i(C_i, X_i, G_i, Z_i, Z_w) \quad (1)$$

We assume  $U_C^i, U_X^i, U_G^i > 0$ ,  $U_Z^i, U_{Z_w}^i < 0$ . Here  $U_C^i$  is the partial derivative with respect to  $C_i$  and the other expressions have a similar meaning. Moreover, we assume that the utility function satisfies the usual conditions such as differentiability, concavity in  $(C, X, G)$ , convexity in  $(Z, Z_w)$  and, in maximization processes, allows for interior solutions.

## 3 First best

### 3.1 Social optimum

A (first best) Pareto optimum may be characterized as a feasible allocation which maximizes a weighted sum of individual utilities, since we can identify states and their representative consumers. Under convexity assumptions, varying the weights provides an easy way to describe the whole set of Pareto optima. The weights reveal how much the planner cares about each individual agent. They may be interpreted as the derivatives of a social utility function having individual utility levels as arguments. Some care must be taken when interpreting the weights along these lines however, as this interpretation implicitly assumes comparability of individual utility levels. Ordinal transformations of individual utility functions do not change the set

of social optimal, but do change the weights attached to each one of these optima.

Let  $\beta = (\beta_1, \beta_2, \dots, \beta_n)$  be the weights attached to the utilities of the representative consumer in each jurisdiction. A (first best) Pareto optimum is a solution of the following problem:

$$\max \sum_{i=1}^n \beta_i U^i(C_i, X_i, G_i, X_i, X_w) \quad (2)$$

subject to the resource constraint

$$\sum_{i=1}^n Y_i \geq \sum_{i=1}^n (C_i + X_i + G_i) \quad (3)$$

and non-negativity conditions.

Under mild non-satiation conditions, the resource constraint (3) is binding at a Pareto optimum.

**Proposition 1** *A first best Pareto optimum is characterized by the following conditions:*

$$\beta_1 U_C^1 = \dots = \beta_i U_C^i = \dots = \beta_n U_C^n \quad (4)$$

$$\frac{U_G^i}{U_C^i} = 1, \quad i = 1, 2, \dots, n \quad (5)$$

$$\frac{U_X^i}{U_C^i} + \frac{U_Z^i}{U_C^i} + \sum_{j=1}^n \frac{U_{Z_w}^j}{U_C^j} = 1, \quad i = 1, 2, \dots, n \quad (6)$$

The proof of this proposition is in the appendix, as well as the proofs of all other propositions.

The three sets of conditions and the resource constraint (3) jointly determine optimal levels of production of all goods.

Conditions (5) and (6) are efficiency conditions which do not depend on the social weights  $\beta$  attached to the various consumers. They state that the consumers' marginal rate of substitution between the three goods should be equal to one, namely their relative production costs. The MRS between the polluting good  $X_i$  and the non-polluting good  $C_i$  takes into account the negative damages it creates, both locally and worldwide, through global warming. These damages are evaluated in terms of the private non-polluting good.

Condition (4) selects a particular Pareto optimum, depending on the chosen social weights attached to individual consumers. It shows that weighted marginal utilities derived from clean consumption should be equalized across countries.

Let us now see how this Pareto optimum may be implemented. We assume that a world regulator is in charge of the control of local and global externalities and of public goods provision. Local governments have no role. Without loss of generality, we assume that clean consumption is not taxed. The central government imposes differentiated pollution taxes  $\theta_i$  ( $i = 1, 2, \dots, n$ ) on consumers in all countries. It finances all public spending  $G_i$  and makes direct positive or negative transfers  $T_{ci}$  to consumers.

Budget constraints for the consumers and the federal government are:

$$C_i + (1 + \theta_i)X_i = Y_i + T_{ci}, \quad i = 1, 2, \dots, n \quad (7)$$

$$\sum_{i=1}^n \theta_i X_i = \sum_{i=1}^n (G_i + T_{ci}) \quad (8)$$

The representative consumer of country  $i$  maximizes utility under the budget constraint (7), taking as given pollution levels  $Z_i$  and  $Z_w$ . The optimality condition is:

$$\frac{U_X^i}{U_C^i} = 1 + \theta_i, \quad i = 1, 2, \dots, n \quad (9)$$

Comparison with (6) shows that optimality requires tax rates

$$\theta_i = -\frac{U_Z^i}{U_C^i} - \sum_{j=1}^n \frac{U_{Z_w}^j}{U_C^j}, \quad i = 1, 2, \dots, n \quad (10)$$

with quantities taken at the optimum. Each tax rate appears as the sum of a local component, equal to the local marginal damage, and a common global one, equal to the sum of all worldwide marginal damages resulting from global warming:

$$\theta_i = \phi_i + \tau, \quad \phi_i = -\frac{U_Z^i}{U_C^i}, \quad \tau = -\sum_{j=1}^n \frac{U_{Z_w}^j}{U_C^j} \quad (11)$$

With these Pigovian tax rates, condition (9) states that the price of the polluting good, for the consumer, is equal to its true marginal cost, including

its own marginal production cost, equal to one, and marginal environmental costs. Optimality also requires proper levels of provision of public goods, according to condition (5). Finally, lump-sum transfers  $T_{ci}$  are used to reach a specific Pareto optimum, characterized by the weights  $\beta_i$ s attached to each country. These transfers may be positive or negative. In the case where first best Pigovian tax receipts are sufficient to finance all first best public spendings,

$$\sum_{i=1}^n \theta_i X_i \geq \sum G_i,$$

the world regulator's budget constraint (8) shows that the sum  $\sum T_{ci}$  of feasible transfers to consumers is positive. All of them may be positive. In the opposite case, at least one consumer must receive a negative transfer.

### 3.2 Decentralization

The analysis of tax rates suggests that the optimal allocation may also be implemented in a more decentralized setting, where a local government imposes a local tax, aimed at correcting the local externality, while a global (carbon) tax is implemented at the world level. We may then assume that the world regulator redistributes the receipts of the carbon tax to local governments rather than to consumers.

Consumer budget constraints are still (7), with  $\theta_i = \phi_i + \tau$ . The consumer pays tax  $\phi_i$  to the local government and tax  $\tau$  to the world regulator. Let  $T_i$  be the transfer to government  $i$ . Local government budget constraints are

$$G_i + T_{ci} = \phi_i X_i + T_i \quad (12)$$

while the world regulator budget constraint is

$$\sum_{i=1}^n T_i = \tau \sum_{i=1}^n X_i \quad (13)$$

Carbon tax receipts are redistributed to local governments. Each of them uses it as well as local tax receipts to finance local public goods provision and lump-sum transfers to its consumers. Nothing prevents transfers  $T_i$  and  $T_{ci}$  from being negative however.

From (7) and (12) we obtain country budget constraints:

$$C_i + G_i + (1 + \tau)X_i = Y_i + T_i, \quad i = 1, 2, \dots, n \quad (14)$$

They can be written as

$$T_i = \tau X_i + (C_i + G_i + X_i - Y_i), \quad i = 1, 2, \dots, n \quad (15)$$

Each transfer to a government includes the redistribution of the world taxes levied on the country and an additional term taking care of the desired redistribution between countries.

The conclusion of this exercise is that differences in local energy taxes do not imply to forsake the uniqueness of the carbon tax, provided that local taxes are optimally designed to counter local externalities. This result is natural in a first best setting where lump-sum transfers may be freely implemented between countries and consumers. The issue is to see whether this result still holds in a second best setting.

## 4 Second best

We now assume that governments are unable to finance public good provision through lump-sum taxation of consumers and must instead resort to distortionary taxation. We follow the Ramsey optimal taxation approach. The new constraint is simply that lump-sum transfers  $T_{ci}$  have to be non-negative.

As was first shown by Sandmo [1975], the presence of externalities modifies the optimal tax scheme. If Pigovian taxation receipts are sufficient to finance public good provision and the desired redistribution is limited, the first best allocation is attainable. Tax receipts are able to finance public goods provision and some positive lump-sum transfers to consumers. If not, additional distortionary taxation is required and we switch to a second best situation.

In our framework, the non-negativity of transfers to consumers takes the following form, which follows from (7) :

$$C_i + (1 + \theta_i)X_i \geq Y_i, \quad i = 1, 2, \dots, n \quad (16)$$

The overall transfer to a country may be defined as in (15).

### 4.1 Second best optimum

A world regulator is again in charge of all public decisions. Formally, the problem is to maximize (2) under the resource constraint (3) and the addi-

tional incentive constraint

$$U_C^i(C_i, X_i, G_i, X_i, X_w)(C_i - Y_i) + X_i U_X^i(C_i, X_i, G_i, X_i, X_w) \geq 0, \quad i = 1, 2, \dots, n \quad (17)$$

This is a so-called primal version of constraint (16), where we use the consumer's optimality condition (9) to eliminate the tax rate.

Assuming that the resource constraint is global amounts to accepting that intercountry transfers  $C_i + G_i + X_i - Y_i$  may be freely implemented. They sum to zero and may be positive or negative.

**Proposition 2** *A second best Pareto optimum with intercountry transfers is characterized by the following conditions:*

$$\beta_1(1 + \mu_1 + \mu_1 H_C^1)U_C^1 = \dots = \beta_n(1 + \mu_n + \mu_n H_C^n)U_C^n \quad (18)$$

$$\frac{1 + \mu_i H_G^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_G^i}{U_C^i} = 1, \quad i = 1, 2, \dots, n \quad (19)$$

$$\begin{aligned} & \frac{1 + \mu_i + \mu_i H_X^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_X^i}{U_C^i} + \frac{1 + \mu_i H_Z^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_Z^i}{U_C^i} \\ & + \sum_{j=1}^n \frac{1 + \mu_j H_{Z_w}^j}{1 + \mu_j + \mu_j H_C^j} \frac{U_{Z_w}^j}{U_C^j} = 1, \quad i = 1, 2, \dots, n \end{aligned} \quad (20)$$

where

$$H_C^i = (C_i - Y_i) U_{CC}^i / U_C^i + X_i U_{XC}^i / U_C^i \quad (21)$$

$$H_G^i = (C_i - Y_i) U_{CG}^i / U_G^i + X_i U_{XG}^i / U_G^i \quad (22)$$

$$H_X^i = (C_i - Y_i) U_{CX}^i / U_X^i + X_i U_{XX}^i / U_X^i \quad (23)$$

$$H_Z^i = (C_i - Y_i) U_{CZ}^i / U_Z^i + X_i U_{XZ}^i / U_Z^i \quad (24)$$

$$H_{Z_w}^i = (C_i - Y_i) U_{CZ_w}^i / U_{Z_w}^i + X_i U_{XZ_w}^i / U_{Z_w}^i \quad (25)$$

The conditions for a Pareto optimum must be adjusted to take into account the impossibility to finance public goods through a lump-sum tax on consumers. Variable  $\mu_i$  is the cost of public funds in country  $i$  or, more precisely, the cost of being unable to levy a lump-sum tax on the representative consumer in this country. It is zero if the world regulator wishes instead to make a *positive* transfer  $T_{ci}$  to this consumer. It might be the case that the world regulator wishes to do so for all consumers. This occurs if, on the one

hand, tax receipts from strictly Pigovian taxes (the first best taxes) prove sufficient to finance all the desired public goods and, on the other hand, the world regulator is satisfied with redistributing the excess as positive lump-sum taxes to all consumers, as he/she does not want to alter too much the world distribution of income. In such a case all  $\mu_i$ s are zero and the second best optimality conditions reduce to the first best ones. This is a very special case however. A more plausible case is the one where some, or all,  $\mu_i$ s are positive. First best conditions must then be adjusted to take into account the costs of public funds.

A particular case is the one where the utility functions (1) are separable and quasi-linear in  $C_i$ . Then  $U_c^i$  equals unity and the cross derivatives are nil which implies  $H_C^i = H_G^i = H_Z^i = H_{Z_w}^i = 0$  and  $H_X^i = X_i U_{XX}^i / U_X^i$ . The first two optimality conditions reduce to

$$\beta_1(1 + \mu_1) = \dots = \beta_n(1 + \mu_n)$$

$$U_G^i = 1 + \mu_i$$

Public goods provision must be pushed in each country to the level where the marginal utility of public goods equals their production cost, here equal to one, augmented by the cost  $\mu_i$  of public funds. Moreover, optimal international redistribution requires the equality of the weighted overall costs of public goods  $1 + \mu_i$ .

In the general non-linear case, the conditions are modified to take into account cross-effects represented by the  $H^i$ s, which are sums of second order elasticities and are standard in the Ramsey approach.

The three sets of conditions appearing in Proposition 2, together with resource constraint (3) and incentive constraints (17) determine the second best optimum. Conditions (19) and (20) are efficiency conditions which do not depend on the social weights  $\beta$ . Conditions (18) select a particular optimum, depending on  $\beta$ .

From these necessary conditions we can retrieve the second best optimal pollution tax  $\theta_i$  and obtain a meaningful decomposition of this tax rate. It follows from (9) that  $U_X^i = (1 + \theta_i)U_C^i$ . From condition (20), we have the following.

**Corollary 3** *In a second best setting with intercountry transfers, the carbon tax is unique. The overall tax on the polluting good may be decomposed in the sum of two local taxes and one global tax:*

$$\theta_i = \phi_i + \psi_i + \tau$$

with

$$\phi_i = -\frac{1 + \mu_i H_Z^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_Z^i}{U_C^i}, \quad \psi_i = \mu_i \frac{H_C^i - H_X^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_X^i}{U_C^i}$$

$$\tau = -\sum_{j=1}^n \frac{1 + \mu_j H_{X_w}^j}{1 + \mu_j + \mu_j H_C^j} \frac{U_{Z_w}^j}{U_C^j}$$

Tax  $\phi_i$  can be seen as a Pigovian tax on polluting the local environment. Tax  $\psi_i$  is a Ramsey tax designed to finance local public good provision. Finally,  $\tau$  is a world carbon tax needed to curb greenhouse gas emissions.

If the optimal second best allocation is such that country  $i$  consumers receive a positive transfer  $T_{ci}$ , the incentive constraint (17) is not binding in this country and the cost of public funds  $\mu_i$  equal to zero. The Pigovian tax  $\phi_i$  is then equal to the value that yields the first best (see (11)), while the Ramsey tax  $\psi_i$  designed to provide additional financing for the public good is equal to zero.

As in the first best case, we may allow for some decentralization of taxation at the local government level. The budget constraints (12) which were relevant in the case of the first best, must be adapted to take into account the local Ramsey taxes  $\psi_i$ . Government  $i$ 's budget constraint now is

$$G_i + T_{ci} = (\phi_i + \psi_i) X_i + T_i, \quad T_{ci} \geq 0 \quad (26)$$

The world regulator collects the carbon tax on consumers and redistributes the receipts as lump-sum transfers  $T_i$  to local governments. The sum of these transfers is positive but some of them may be negative. If local Pigovian taxation receipts  $\phi_i X_i$  are large, or if redistribution favors country  $i$ , taking the form of a large positive  $T_i$ , the country will not have to rely too much on the supplementary distortive tax to finance the provision of public goods. The cost  $\mu_i$  of public funds will be low. It may even be zero, in which case the supplementary Ramsey tax  $\psi_i$  becomes zero and the government may make a positive transfer  $T_{ci}$  to its consumers.

The striking thing here is that the carbon tax  $\tau$  is unique: it is the same for all states involved. This result would seem natural in a first best setting. It is much more surprising in a second best framework, where optimal rules are usually quite different from what they would be in a first best. The virtues of a unique carbon tax and its ability to provide proper incentives are much less compelling when specific constraints preexist and unavoidably introduce many distortions in the economy. Differentiated carbon prices might then



appear as a useful compromise between efficiency and equity. We do assume the feasibility of lump-sum international transfers, which allows to take into account equity concerns. This does not free us from the constraints of a second best world however.

## 4.2 Decentralization through a world carbon price and intercountry transfers

To put our result in perspective, we now show that the second best optimum may be obtained in a decentralized way. The world regulator sets the world carbon tax and lump-sum transfers to local governments. Each of these is then free to deal as it wishes with local externalities and distortionary taxation needed to finance the provision of local public goods.

A close examination of the determination of a second best Pareto optimum suggests a two stage solution method. Let us rewrite the problem

$$\max_{C, X, G, Z_w} \sum_{i=1}^n \beta_i U^i(C_i, X_i, G_i, X_i, Z_w)$$

$$U_C^i(C_i, X_i, G_i, X_i, Z_w)(C_i - Y_i) + X_i U_X^i(C_i, X_i, G_i, X_i, Z_w) \geq 0, \quad i = 1, \dots, n \quad (27)$$

$$\sum_{i=1}^n X_i = Z_w \quad (28)$$

$$\sum_{i=1}^n (C_i + X_i + G_i) \leq \sum_{i=1}^n Y_i \quad (29)$$

We may check that this problem is equivalent to a problem where we introduce new variables  $\tau$  and  $T_i$ ,  $i = 1, \dots, n$  and substitute constraints

$$C_i + G_i + (1 + \tau)X_i = Y_i + T_i, \quad i = 1, 2, \dots, n \quad (30)$$

$$\sum_{i=1}^n T_i \leq \tau Z_w. \quad (31)$$

to constraint (29).

Taking into account (28), constraints (30) and (31) imply (29). Reciprocally, for any arbitrary  $\tau$ , consider a triplet  $(C, X, G)$  satisfying (29). Define

$T_i = C_i + G_i + (1 + \tau)X_i - Y_i$ ,  $i = 1, 2, \dots, n$ . Summing implies  $\sum T_i - \tau Z_w = \sum (C_i + G_i + X_i - Y_i)$  which is negative as  $(C, X, G)$  satisfies (29). Thus constraint (31) is satisfied.

Let us now consider the maximization of social welfare under constraints (28), (30) and (31). We may solve this problem in two stages. For given  $Z_w$ ,  $\tau$  and  $T$ , we first look, for all  $i$ , for  $C_i$ ,  $G_i$ ,  $X_i$  which maximize  $U^i(C_i, X_i, G_i, X_i, Z_w)$  under constraints (27) and (30). This yields functions

$$C_i = \chi^i(\tau, T_i, Z_w), \quad X_i = \xi^i(\tau, T_i, Z_w), \quad G_i = \gamma^i(\tau, T_i, Z_w)$$

and the indirect utility function

$$W^i(\tau, T_i, Z_w)$$

In the second stage, we solve the following problem<sup>3</sup>:

$$\max_{\tau, T, Z_w} \sum_{i=1}^n \beta_i W^i(\tau, T_i, Z_w)$$

under constraints (29) and (31), (29) being now written as

$$\sum_{i=1}^n \xi^i(\tau, T_i, Z_w) = Z_w$$

This procedure may be interpreted as a decentralized implementation of a second best optimum. The world regulator controls carbon emissions, through a permit market or a carbon tax. It is clearer to assume the former. The world regulator chooses a maximum amount  $Z_w$  of emissions, sells off the permits to local governments and distributes the proceeds as lump-sum transfers to local governments. These transfers may be negative in order to further subsidize other countries. A unique carbon price is thus imposed at the world level. Each local government is still unable to finance its public goods provision through lump-sum taxation. But it is left free to choose its own way to levy funds and regulate local externalities. A subsidiarity principle is thus in effect.

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<sup>3</sup>The carbon price  $\tau$  is not a choice variable since it is determined endogenously through the equilibrium of the permits market. However, as stressed by Shiell [2003], it is mathematically equivalent to treat it as a choice variable subject to the constraint that the permits market clears.

Local governments are passive in the sense that they take as given the price  $\tau$  of permits as well as total emissions  $Z_w$  and transfers  $T_i$ s. Their problem is to maximize the utility of their representative consumer under their country budget constraint (14) and their incentive constraint (17). We do not make explicit the precise way they will implement this solution. The world regulator only takes into account the indirect utility function  $W^i(\tau, T_i, Z_w)$  of each country and the associated demand of permits  $\xi^i(\tau, T_i, Z_w)$ .

**Proposition 4** *A second best optimum with intercountry transfers may be implemented by a world regulator controlling the total amount of emissions, imposing a unique carbon price and implementing intercountry lump-sum transfers.*

This result shows how effective a unique carbon price remains in a framework where each country faces second best constraints. Control of total emissions and intercountry transfers are the sole international coordination required to reach a social optimum. We also note that the proof does not depend on the form of the incentive constraints and is therefore quite general.

This proof provides a deeper explanation of the uniqueness of the carbon tax. Our first proof relies on the inspection of optimality conditions. The decentralization described in this section shows that this uniqueness property is in fact the consequence of a subsidiarity principle. The world regulator does not have to take into account the precise way in which each country regulates local externalities and finances the provision of its public goods. He may trust that this is achieved efficiently and only care about the global climatic externality and the proper distribution of permits. In some sense, the international regulation of the climatic externality remains a first best problem.

### 4.3 Second best without intercountry transfers

We now assume, as in Shiell [2003], that for political economy reasons, local governments are not ready to accept a smaller transfer than the amount of carbon taxes they are paying to the world regulator. Each local government actually receives a positive transfer from the world regulator, as the country level of emissions is always positive. There are no other intercountry transfers. Each country then has to satisfy constraint

$$C_i + X_i + G_i = Y_i, \quad i = 1, 2, \dots, n \quad (32)$$

The problem is to maximize (2) under the resource constraints (32) and the additional incentive constraints (17).

**Proposition 5** *A second best Pareto optimum without intercountry transfers is characterized by the following conditions:*

$$\beta_i \{ (1 + \mu_j + \mu_j H_X^j) U_X^j + (1 + \mu_j H_Z^j) U_Z^j - (1 + \mu_j + \mu_j H_C^j) U_C^i \} = - \sum \beta_j (1 + \mu_j H_Z^j) U_{Zw}^j, \quad i = 1, 2, \dots, n \quad (33)$$

$$\frac{1 + \mu_i H_G^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_G^i}{U_C^i} = 1, \quad i = 1, 2, \dots, n \quad (34)$$

$$\sum_{j=1}^n \frac{(1 + \mu_j H_{Zw}^j) U_{Zw}^j}{(1 + \mu_j + \mu_j H_C^j) U_C^j - (1 + \mu_j + \mu_j H_X^j) U_X^j - (1 + \mu_j H_Z^j) U_Z^j} = 1 \quad (35)$$

The three sets of conditions appearing in Proposition 5, together with the resource constraints (3) and incentive constraints (17) determine the second best optimum without transfers. Condition (34) governing the provision of public goods is the same as the one which appears in the second best optimum with intercountry transfers (condition (19)). Condition (35) is implied by the set of similar conditions (20) appearing in the case with transfers. Indeed, (20) may be written as

$$\frac{(1 + \mu_i + \mu_i H_X^i) U_X^i + (1 + \mu_i H_Z^i) U_Z^i - (1 + \mu_i + \mu_i H_C^i) U_C^i}{(1 + \mu_i + \mu_i H_C^i) U_C^i} = - \sum_{j=1}^n \frac{1 + \mu_j H_{Zw}^j}{1 + \mu_j + \mu_j H_C^j} \frac{U_{Zw}^j}{U_C^j}, \quad i = 1, 2, \dots, n \quad (36)$$

Dividing through by the common value of the LHS yields (35).

Consider now a particular allocation in the set of second best optima with intercountry transfers, where all transfers happen to be zero. Such an equilibrium usually exists and is unique. By definition it satisfies constraints (32) and therefore all constraints entering the definition of a second best optimum without intercountry transfers. We just showed that it satisfies all the optimality conditions for such an optimum. We thus confirm the rather natural property that, in general, one of the second best optima without

intercountry transfers is a particular second best optimum with intercountry transfers. Both hypersurfaces in the space of utility levels will be tangent. This property has been stressed by Shiell [2003].

Other second best optimum allocations without transfers are not second best optimum allocations with transfers. As we now show, this means that differentiated carbon prices will be necessary to support them.

**Corollary 6** *In a second best setting without intercountry transfers, the carbon tax is generally not unique. The overall tax on the polluting good can be decomposed into three country-specific taxes:*

$$\theta_i = \phi_i + \psi_i + \tau_i$$

with

$$\begin{aligned}\phi_i &= -\frac{1 + \mu_i H_Z^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_Z^i}{U_C^i}, & \psi_i &= \mu_i \frac{H_C^i - H_X^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_X^i}{U_C^i} \\ \tau_i &= -\frac{1}{\beta_i (1 + \mu_i + \mu_i H_C^i) U_C^i} \sum \beta_j (1 + \mu_j H_{X_w}^j) U_{Z_w}^j\end{aligned}$$

The expressions of the Pigovian and the Ramsey tax rates  $\phi_i$  and  $\psi_i$  are the same than in the case with intercountry transfers. The carbon tax  $\tau_i$  is now country-specific. More precisely, we have

$$\beta_1 (1 + \mu_1 + \mu_1 H_C^1) U_C^1 \tau_i = \dots = \beta_n (1 + \mu_n + \mu_n H_C^n) U_C^n \tau_n \quad (37)$$

Carbon prices are not equalized worldwide. Only weighted carbon prices are, the weight of country  $i$  being  $\beta_i (1 + \mu_i + \mu_i H_C^i) U_C^i$ . This generalizes the similar relation which underlines the results of Chichilnisky and Heal [1994]. As we take into account the provision of public goods, a new element  $1 + \mu_i + \mu_i H_C^i$  appears which takes into account the cost of public funds in the country.

We may then restate the core of the Chichilnisky and Heal [1994] argument. In the case where intercountry transfers are feasible, condition (18) in Proposition 2 states that country weights are equalized. It follows that carbon prices, and abatement costs in the Chichilnisky and Heal [1994] set-up, are equalized. In the absence of intercountry transfers, the weights are not equalized and neither are the optimal carbon prices.

We may assume that social weights  $\beta$  reflect the world regulator's aversion to inequality, which would automatically follow if we deduced them from the maximization of a symmetric international social utility function. A *poorer*

country is then characterized by a higher  $\beta_i$ . This leads to reduction of inequality, but probably not its disappearance. A poorer country presumably remains poorer in the double sense that it has both a lower level of consumption, and therefore a higher marginal utility  $U_C^i$  of consumption, and a higher cost of public funds, and therefore a higher  $1 + \mu_i + \mu_i H_C^i$  coefficient. We may thus safely assume that a poorer country is characterized by a larger weight  $\beta_i(1 + \mu_i + \mu_i H_C^i)U_C^i$ . The carbon price  $\tau_i$  must then be lower in a poorer country. A smaller environmental effort must be required from poor countries, which is simply another part of the worldwide social optimization.

## 5 Conclusion

It has been shown that the presence of public goods, restrictions on the redistributions of local tax revenues to local consumers, and the addition of a local component to the damage caused by global emissions does in principle not aggravate the problem of addressing the design of a second best optimal global pollution tax. An appropriate decomposition of the pollution tax can deal with the local aspect of pollution and with the constraint that only non-negative transfers can be given to local consumers. Hence, the global externality can be dealt with at the global level, through a uniform tax or a system of tradable permits. Nevertheless it is required that at the government levels international transfers can still be implemented. And it cannot be hoped that this problem can be solved. Indeed, it is uncertain that governments will accept negative lump-sum transfers and even to receive less than what the country paid as carbon taxes. Chichilnisky and Heal [1994] have stressed that the impossibility of international redistribution should lead to reject the principle of equalizing worldwide abatement costs, that is having a unique world carbon price. If we introduce constraints on intercountry transfers into our model, a unique carbon price would not allow to reach the desired second best optimum. International differentiation of the price of carbon sometimes provides a useful instrument to reach a more equitable allocation. However, when it comes to climate change international transfers (whether monetary or technological or in terms of allocating permits) are in the picture already. The international community is aware of the fact that in reaching international agreements side payments are unavoidable.

## A Proof of Proposition 1

Let  $\lambda$  be the Lagrange multiplier associated to the resource constraint. Necessary conditions are:

$$\beta_i U_C^i = \lambda, \quad i = 1, 2, \dots, n \quad (38)$$

$$\beta_i U_G^i = \lambda, \quad i = 1, 2, \dots, n \quad (39)$$

$$\beta_i (U_X^i + U_Z^i) + \sum_{j=1}^n \beta_j U_{Z_w}^j = \lambda, \quad i = 1, 2, \dots, n \quad (40)$$

Conditions (38) and (39) yield condition (5) in Proposition 1.

Dividing both sides of (40) by  $\lambda$ , and noting from (38) that  $\lambda$  may be taken equal to  $\beta_i U_C^i$  or to any of the  $\beta_j U_C^j$  yields

$$\frac{U_X^i}{U_C^i} + \frac{U_Z^i}{U_C^i} + \sum_{j=1}^n \frac{U_{Z_w}^j}{U_C^j} = 1, \quad i = 1, 2, \dots, n$$

which is condition (6) in Proposition 1.

## B Proof of Proposition 2 and Corollary 3

Let  $\lambda$  and  $\tilde{\mu}_i = \mu_i \beta_i$  be the Lagrange multipliers associated to constraints (3) and (17).  $\mu_i$  will be interpreted as the cost of public funds in country  $i$ .

Assuming an interior solution for  $(C_i, X_i, G_i, X_i X_w) > 0$ , we can write the optimality conditions as follows:

$$\beta_i U_C^i \{1 + \mu_i + \mu_i H_C^i\} = \lambda \quad (41)$$

$$\beta_i U_G^i \{1 + \mu_i H_G^i\} = \lambda \quad (42)$$

$$\beta_i U_X^i (1 + \mu_i + \mu_i H_X^i) + \beta_i U_Z^i (1 + \mu_i H_Z^i) + \sum_{j=1}^n \beta_j U_{X_w}^j (1 + \mu_j H_{X_w}^j) = \lambda \quad (43)$$

(41) and (42) yield condition (19) in Proposition 2.

Dividing (43) by  $\lambda$  and choosing from (41), in each place, the appropriate expression for  $\lambda$  leads to (20) in Proposition 2. This equation may be written as

$$\left(1 + \mu_i \frac{H_X^i - H_C^i}{1 + \mu_i + \mu_i H_C^i}\right) \frac{U_X^i}{U_C^i} + \frac{1 + \mu_i H_Z^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_Z^i}{U_C^i} + \sum_{j=1}^n \frac{1 + \mu_j H_{Z_w}^j}{1 + \mu_j + \mu_j H_C^j} \frac{U_{Z_w}^j}{U_C^j} = 1$$

or

$$\frac{U_X^i}{U_C^i} - 1 = -\mu_i \frac{H_X^i - H_C^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_X^i}{U_C^i} - \frac{1 + \mu_i H_Z^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_Z^i}{U_C^i} - \sum_{j=1}^n \frac{1 + \mu_j H_{Z_w}^j}{1 + \mu_j + \mu_j H_C^j} \frac{U_{Z_w}^j}{U_C^j}$$

which is Corollary 3.

## C Proof of Proposition 5 and Corollary 6

Let  $\lambda_i$  and  $\tilde{\mu}_i = \mu_i \beta_i$  be the Lagrange multipliers associated to constraints (32) and (17). Optimality conditions are, for  $i = 1, 2, \dots, n$ ,

$$\beta_i (1 + \mu_i + \mu_i H_C^i) U_C^i = \lambda_i \quad (44)$$

$$\beta_i (1 + \mu_i H_G^i) U_G^i = \lambda_i \quad (45)$$

$$\beta_i \left\{ (1 + \mu_i + \mu_i H_X^i) U_X^i + (1 + \mu_i H_Z^i) U_Z^i \right\} = \lambda_i - \sum_{j=1}^n \beta_j (1 + \mu_j H_{Z_w}^j) U_{Z_w}^j \quad (46)$$

(44) and (45) imply (34) in Proposition 5.

From (44) and (46) we have

$$\begin{aligned} \beta_i \left\{ (1 + \mu_i + \mu_i H_X^i) U_X^i + (1 + \mu_i H_Z^i) U_Z^i - (1 + \mu_i + \mu_i H_C^i) U_C^i \right\} = \\ - \sum_{j=1}^n \beta_j (1 + \mu_j H_{Z_w}^j) U_{Z_w}^j, \quad i = 1, 2, \dots, n \end{aligned} \quad (47)$$

The RHS does not depend on  $i$  so that all the LHS are equal for all  $i$ . Dividing through by this common value, and in particular dividing each term of the sum by the  $j$ -indexed expression of the LHS yields

$$1 = \sum_{j=1}^n \frac{(1 + \mu_j H_{Z_w}^j) U_{Z_w}^j}{(1 + \mu_j + \mu_j H_C^j) U_C^j - (1 + \mu_j + \mu_j H_X^j) U_X^j - (1 + \mu_j H_Z^j) U_Z^j}$$



which is (35) in Proposition 5.

Condition (47) may be written as

$$\left(1 + \mu_i \frac{H_X^i - H_C^i}{1 + \mu_i + \mu_i H_C^i}\right) \frac{U_X^i}{U_C^i} + \frac{1 + \mu_i H_Z^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_Z^i}{U_C^i} - 1 =$$

$$- \frac{1}{\beta_i (1 + \mu_i + \mu_i H_C^i) U_C^i} \sum \beta_j (1 + \mu_j H_Z^j) U_{Z_w}^j$$

or

$$\frac{U_X^i}{U_C^i} - 1 = - \frac{1 + \mu_i H_Z^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_Z^i}{U_C^i} - \mu_i \frac{H_X^i - H_C^i}{1 + \mu_i + \mu_i H_C^i} \frac{U_X^i}{U_C^i}$$

$$- \frac{1}{\beta_i (1 + \mu_i + \mu_i H_C^i) U_C^i} \sum \beta_j (1 + \mu_j H_Z^j) U_{Z_w}^j$$

which is Corollary 6.

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